Diffusion and entanglement of a kicked particle in an infinite square well under frequent measurements

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We investigate the dynamics of a kicked particle in an infinite square well undergoing frequent measurements of energy. For a large class of periodic kicking forces, constant diffusion is found in such a non-Kolmogorov-Arnol'd-Moser system. The influence of a phase shift of the kicking potential on the short-time dynamical behavior is discussed. The general asymptotical measurement-assisted diffusion rate is obtained. The entanglement between the particle and the measuring apparatus is investigated. There exist two distinct dynamical behaviors of entanglement. The bipartite entanglement between the system of interest and the whole spin of the measuring apparatus grows with the kicking steps and it gains a larger value for a more chaotic system. However, the partial entanglement between the system of interest and the partial spin of the measuring apparatus decreases with the kicking steps. The relation between the entanglement and quantum diffusion is also analyzed.

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I. INTRODUCTION

One of the significant consequences of quantum mechanics is that the measurement unavoidably disturbs the measured system. This is particularly revealed by the so-called Zeno and anti-Zeno effects [1–3]. The Zeno (anti-Zeno) effect refers to the inhibition (acceleration) of the evolution when one attempts to observe it, and can be regarded as two particular consequences of the disturbance of the observed system caused by quantum measurement. The first experiment on the quantum Zeno effect was proposed by Cook [4], and realized by Itano *et al.* [5] using coherent Rabi oscillations in a three-level atom. The opposite phenomenon, the anti-Zeno effect (decay acceleration by frequent measurements) was recently discovered by Kofman and Kurizki [2,3]. In the theoretical and numerical investigations of quantum chaotic systems, similar predictions have been presented [6–10].

Dynamical localization, i.e., the quantum mechanical suppression of classical chaotic diffusion, was first discovered by Casati *et al.* [11] in their investigation of the kicked rotor (standard map), which can be understood as a dynamical version of Anderson localization [12]. Quantum localization emerges due to quantum interference, which can be destroyed by noise and interactions with the environment, i.e., decoherence. Many theoretical and experimental studies have shown that even a small amount of noise demolishes localization [13–16]. Quantum measurements can be regarded as another type of coupling to the "environment," i.e., the measurement apparatus [17]. Kaulakys and Gontis have shown that, in the case of the kicked rotor, a diffusive behavior can emerge even in the quantum case, if a projective measurement is performed after every kick [6]. Facchi *et al.* have found that projective measurements provoke diffusion even when the corresponding classical dynamics is regular [8]. Dittrich and Graham have studied the kicked rotor coupled to macroscopic systems acting as continuous measuring devices with limited time resolution and found the diffusive energy growth [18]. Most previous work concerning the anti-Zeno effect in quantum chaos has been concentrated on quantum systems whose classical counterparts obey the Kolmogorov-Arnol'd-Moser (KAM) theorem. So it is interesting to investigate the influence of frequent measurements on the dynamical behavior of the non-KAM system, such as a kicked particle in an infinite square well [19]. In this paper, we investigate the dynamics of a kicked particle in an infinite square well undergoing frequent measurements of energy. It is found that, for a large class of periodic kicking forces, the dynamical behaviors exhibit diffusion with constant rate in such a non-KAM system. Then, we investigate how the ratio of the well width and the kicking field wavelength affect the diffusion of energy in the present situations. It is shown that not only increasing the kicking field strength but also increasing the ratio of the well width and the kicking field wavelength can enhance the diffusion of energy in such a non-KAM system undergoing repeated measurements.

Recently, much attention has also been paid to entanglement in the quantum chaotic systems. Several authors have studied the entanglement in coupled kicked tops or the Dicke model [20–32], and found it has a manifestation of chaotic behavior. It has been demonstrated that classical chaos can lead to substantially enhanced entanglement and it has been shown that entanglement provides a useful characterization of quantum states in higher-dimensional chaotic systems. For the system of coupled kicked tops, it has been clarified that two initially separable subsystems can get entangled in a nearly linear rate depending on the intrinsic chaotic properties, and their entanglement eventually reaches saturation [21–24,26,27]. It has also been elucidated that the increment of the nonlinear parameter of coupled kicked tops does not accelerate the entanglement production in the strongly chaotic region [26]. For a single kicked top composed of collec-

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tive spin $\frac{1}{2}$, it has been suggested that entanglement can be regarded as a signature of quantum chaos. Both bipartite entanglement and pairwise entanglement in the spins have been considered and it has been revealed that bipartite entanglement is enhanced in the chaotic region; nevertheless pairwise entanglement is suppressed [29]. Most previous work studied how the intrinsic dynamical properties of the quantum chaotic systems affect the entanglement between the subsystems. It may be more natural to explore how the entanglement between the system of interest and its surrounding environment or measuring apparatus affects the chaotic behavior, such as the diffusion behavior. Here, we show that the diffusion of the kicked particle in an infinite square well undergoing frequent measurements of energy is closely related with the entanglement of the particle and the measuring apparatus. It is found that there exist two distinct dynamical behaviors of entanglement. The bipartite entanglement (defined as entanglement between the particle and the whole degree of freedom of the measuring apparatus in this paper) grows with the kicking steps and it gains a larger value for a more chaotic system. However, the partial entanglement between pairs (specifically defined as entanglement between the particle and the partial degree of freedom of the measuring apparatus in this paper) decreases with increasing number of kicking steps. It is very desirable to investigate the asymptotical behavior of the bipartite entanglement or the partial entanglement between pairs. Can the bipartite entanglement reach saturation? In this paper, these are still the open questions.

This paper is organized as follows. In Sec. II, we investigate the diffusion in non-KAM systems. The dynamics of a kicked particle in an infinite square well undergoing frequent measurements of energy is studied in detail. It is found that, for a large class of periodic kicking forces, the dynamical behaviors exhibit diffusion with constant rate in such a non-KAM system. Then, we investigate how the ratio of the well width and the kicking field wavelength affects the diffusion of energy in the present situations. It is shown that not only increasing the kicking field strength but also increasing the ratio of the well width and the kicking field wavelength can enhance the diffusion of energy in such a non-KAM system undergoing repeated measurements. In Sec. III, we focus our attention on the entanglement between this quantum chaotic system and the measuring apparatus by exploring the relative entropy of entanglement. We investigate how the inherent quantum chaos affects the entanglement between the particle and measuring apparatus. Two distinct dynamical behaviors of entanglement are revealed. The bipartite entanglement grows with the kicking steps. It increases with a higher rate for the more chaotic system in the short time. However, the partial entanglement between the particle and the partial spins of the measuring apparatus decreases with the kicking steps. Some conclusive remarks and brief discussion about the experimental verification of measurement-induced diffusion in the quantum dot are given in Sec. IV.

II. DIFFUSION OF THE KICKED PARTICLE IN AN INFINITE SQUARE WELL UNDERGOING FREQUENT MEASUREMENTS OF ENERGY

In this section, we investigate the diffusion of the kicked particle in an infinite square well undergoing frequent measurements of energy. The Hamiltonian investigated here is described by

$$H = \frac{p^2}{2} + V_0(x) + V(x)\sum_{l=-\infty}^{\infty}\delta(t-lT), \qquad (1)$$

where the potential $V_0(x)$ is the confining infinite square well potential of width π, centered at the position $\pi/2$. $V(x)$ is the external potential. Since the two hard walls destroy the analyticity of the potential, the KAM scenario breaks down in the system (1). In Ref. [19], the authors have studied quantum chaos of the system (1) with $V(x)=k\cos(x+\alpha)$, where α is a phase shift. It was shown that, for a small perturbation K $(=kT)$, the classical phase space displays a stochastic web structure, and the diffusion rate scales as $D\propto K^{2.5}$. However, in the large-K regime, $D\propto K^2$. Quantum mechanically, they observed that the quasieigenstates are power-law localized for small K and extended for large K. In what follows, we investigate the evolution of system (1) interrupted by quantum measurements, in the following sense: the system evolves under the action of the free Hamiltonian $P^2/2+V_0(x)$ for $(N-1)T+t'<t<NT$ $(0<t'<T)$, undergoes a kick at $t=NT$, evolves again freely. Then a measurement of the energy E is acted on the system at $t=NT+t'$. The evolution of the density matrix can be written as follows:

$$\rho_{NT+t'} = \mathcal{L}^N \rho_{t'},$$

$$\mathcal{L}\rho = \sum_{n=1}^{\infty} U_{free}(t')U_{kick}U_{free}(T-t')|n\rangle\langle n|\rho|n\rangle\langle n| \times U_{free}^{\dagger}(T-t')U_{kick}^{\dagger}U_{free}^{\dagger}(t'), \qquad (2)$$

where $\rho_{NT+t'}$ represents the density matrix of the particle at the time $NT+t'$, $|n\rangle$ is the nth eigenstate of the nonperturbed Hamiltonian, and

$$U_{kick} = \exp[-iV(x)/\hbar], \quad U_{free}(t) = \exp\left(-i\frac{p^2 t}{2\hbar}\right),$$

$$\langle x|n\rangle = \sqrt{\frac{2}{\pi}}\sin(nx), \quad x\in[0,\pi],\ n=1,2,\ldots. \qquad (3)$$

From Eq. (2), we can derive the occupation probabilities $P_n(N)\equiv\langle n|\rho_{NT+t'}|n\rangle$ which are governed by

$$P_n(N) = \sum_m Z_{nm}P_m(N-1), \qquad (4)$$

where

$$Z_{nm} = |\langle n|U_{kick}|m\rangle|^2 \qquad (5)$$

are the transition probabilities. In Fig. 1, we display the evolution of occupation probabilities governed by Eqs. (4) and (5) with the kicking potential $V(x)=k\cos(x+1)$ for different values of k. It is clearly shown that the particle can be driven by the external periodic field from the ground level to the higher excited levels. In what follows, we will derive an analytical expression for the diffusion rate defined by $D\equiv(E_N-E_0)/N$, where E_N is the expected value of energy of

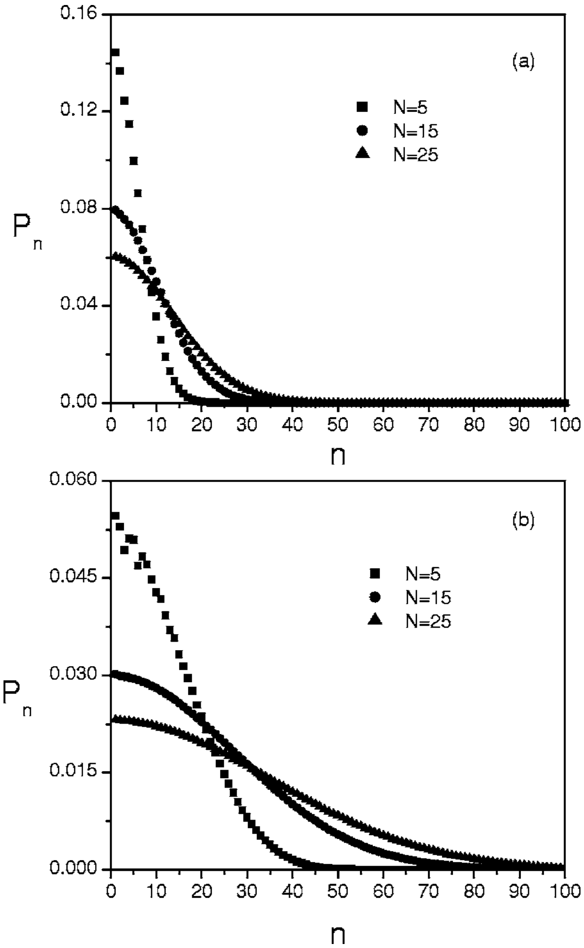

FIG. 1. The occupation probabilities P_n of the particle undergoing the Nth kick and the Nth projective measurement of the energy are depicted for different values of k. The particle is initially in the ground state and the kicking potential $V(x)=k\cos(x+1)$ is chosen as an illustration. $k=$(a) $4\hbar$, and (b) $10\hbar$.

the particle at the time $NT+t'$. By making use of Eqs. (4) and (5), we can obtain

$$\begin{aligned} E_N &= \sum_{n=1}^{\infty} \frac{\hbar^2 n^2}{2} P_n(N) \\ &= E_{N-1} + \frac{1}{2\pi}\int_0^{\pi} (V')^2 dx \\ &\quad - \frac{1}{2\pi}\sum_{m=1}^{\infty} \int_0^{\pi} (V')^2 \cos(2mx)dx\, P_m(N-1), \end{aligned} \tag{6}$$

where V' denotes the first-order derivative of $V(x)$ upon x. From Eq. (6), we can derive a sufficient condition related to the diffusion with constant rate. If $V(x)$ satisfies the following expression:

$$(V')^2 = a_0 + \sum_{m=1}^{\infty} a_m \sin(2mx), \tag{7}$$

where a_m $(m=0,1,2,\ldots)$ are the expansion coefficients which should ensure a positive definite value of the function

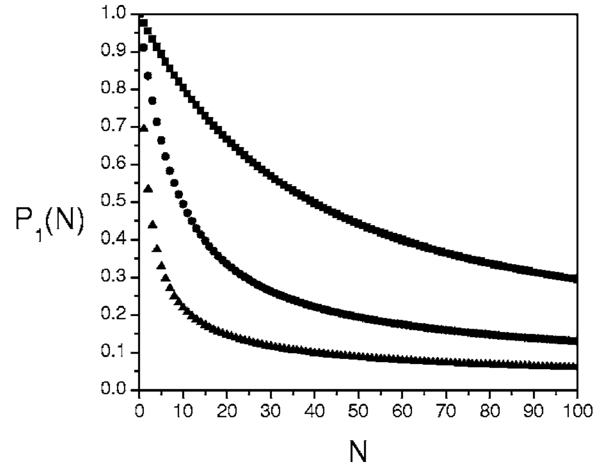

FIG. 2. The ground occupation probabilities P_1 of the particle at the time $NT+t'$ are plotted as a function of N for different values of k. The particle is initially in the ground state and the kicking potential $V(x)=k\cos(x+1)$ is chosen as an illustration. Solid square, $k=0.5\hbar$; solid circle, $k=\hbar$; solid triangle, $k=2\hbar$.

$a_0+\sum_{m=1}^{\infty} a_m \sin(2mx)$, the diffusion rate D is the constant $a_0/2$. As an illustration, we consider the case with $V(x)=k\cos(x+\pi/4)$, which obviously satisfies Eq. (7) and the corresponding expansion coefficients are $a_0=a_1=k^2/2$ ($a_m=0$ for $m>1$). Then, in this case, iterating Eq. (6), we can obtain

$$E_N = E_0 + \frac{k^2 N}{4}, \tag{8}$$

which leads to $D=k^2/4$. For $V(x)=k\cos(x+\alpha)$, we have

$$E_N = E_{N-1} + \frac{k^2}{4} + \frac{k^2\cos(2\alpha)}{8} P_1(N-1). \tag{9}$$

The above equation shows that the increase of energy after every step is closely related to the ground state population of the previous step. Equation (9) also implies that the phase shift α plays a significant role in the short-time dynamical behavior of the kicked particle in this situation. The diffusion will be enhanced or suppressed by adjusting the sign of $\cos(2\alpha)$. However, since $\lim_{N\to\infty} P_1(N)=0$, the energy diffusion is asymptotically linear and independent of α for any values of k. In Fig. 2, the values of P_1 at the time $NT+t'$ are plotted as a function of N for different values of k. It is shown that $P_1(N)$ decreases with increasing N, and the larger the value of k, the more rapid the decay of the particle from the ground state. Therefore, according to Eq. (9), we know that the corresponding diffusion rate of the nonunitary evolution governed by Eq. (2) with $V(x)=k\cos(x+1)$ increases with N and eventually converges to $k^2/4$.

In Ref. [33], the authors have investigated the quantum chaotic dynamics of the system (1) with $V(x)=\cos(2Rx)$, where R is a ratio of two length scales, namely, the well width and the kicking field wavelength. If R is a noninteger the dynamics is non-KAM. It was shown that time-evolving states exhibit considerable R dependence, and tuning R to enhance classical diffusion can lead to significantly larger quantum diffusion for the same field strengths. It is interesting to investigate how the values of R affect the diffusion of energy in the present situations. Substituting $V(x) = k\cos(2Rx)$ into Eq. (6), we can obtain

$$E_N = E_{N-1} + k^2R^2 - \frac{k^2R\sin(4R\pi)}{4\pi} + \frac{k^2R^3\sin(4R\pi)}{\pi}\sum_{m=1}^{\infty}\frac{1}{4R^2-m^2}P_m(N-1). \quad (10)$$

The above equation implies that not only increasing the kicking field strength k but also increasing the ratio R of the well width and the kicking field wavelength can enhance the diffusion of energy in such a non-KAM system undergoing repeated measurements. To elucidate it, we merely need to consider three different cases listed in the following. (1) When R is integer or half integer (i.e., $\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots$), Eq. (10) can be replaced by

$$E_N = E_{N-1} + k^2R^2 + \frac{k^2R^2}{2}P_{2R}(N-1), \quad (11)$$

which is very similar to Eq. (9). (2) When $R=(2j+1)/4$ $(j=0,1,2,\ldots)$., Eq. (10) can be rewritten as

$$E_N = E_{N-1} + k^2R^2, \quad (12)$$

which implies that the diffusion rate is a constant k^2R^2. (3) For any other values of R, the diffusion behavior becomes very complex. In what follows, we analyze both the short-time dynamical behavior and the asymptotical dynamical behavior. For $V(x)=k\cos(2Rx)$, the transition matrix $\langle n|U_{kick}|m\rangle$ can be expressed as

$$\langle n|U_{kick}|m\rangle = J_0(k/\hbar)(\delta_{n,m}-\delta_{n,-m}) + \frac{1}{\pi}\sum_{j=1}^{\infty}(-i)^jJ_j(k/\hbar) \times[C_j(n-m)-C_j(n+m)], \quad (13)$$

where $J_n(k/\hbar)$ is the Bessel function and

$$C_j(l) = \begin{cases} \frac{4(-1)^l jR\sin(2jR\pi)}{4j^2R^2-l^2} & \text{for } 2jR \neq |l|, \\ \pi & \text{for } 2jR = |l|. \end{cases} \quad (14)$$

In Fig. 3, the decay from the initial ground state of the particle is depicted for different values of R. It is shown that the larger the value of R, the more rapid the decay of the particle from the ground state.

If the particle is initially in the ground state, i.e., $P_n(0) = \delta_{n,1}$, we have $P_m(N-1)=(Z^{N-1})_{m,1}$, where $Z\equiv Z_{l_1,l_2}$ is the transition probabilities matrix defined by Eq. (5). Substituting it into Eq. (10), and iterating it, we can obtain

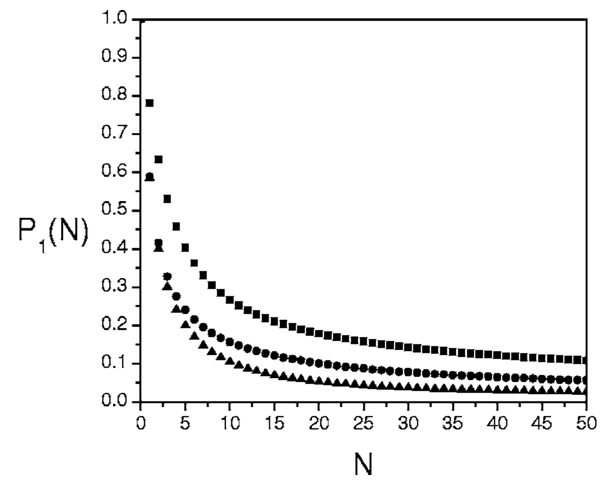

FIG. 3. The ground occupation probabilities P_1 of the particle at the time $NT+t'$ are plotted as a function of N for different values of R. The particle is initially in the ground state and the kicking potential $V(x)=k\cos(2Rx)$ is chosen as an illustration, $k=\hbar$. Solid square, $R=\pi/4$; solid circle, $R=\pi/2$; solid triangle, $R=\pi$.

$$E_N = E_0 + Nk^2R^2 - \frac{Nk^2R\sin(4R\pi)}{4\pi} + \frac{k^2R^3\sin(4R\pi)}{\pi}\sum_{i=0}^{N-1}\sum_{m=1}^{\infty}\frac{1}{4R^2-m^2}(Z^i)_{m,1}$$
$$= E_0 + Nk^2R^2 - \frac{Nk^2R\sin(4R\pi)}{4\pi} + \frac{k^2R^3\sin(4R\pi)}{\pi}\sum_{m=1}^{\infty}\frac{1}{4R^2-m^2}\left(\frac{1-Z^N}{1-Z}\right)_{m,1}. \quad (15)$$

In deriving Eq. (15), we have used the formulation

$$\sum_{i=0}^{N-1}(Z^i)_{m,1} = \left(\frac{1-Z^N}{1-Z}\right)_{m,1}. \quad (16)$$

It is not difficult to verify that

$$\lim_{N\to\infty}\left(\frac{1-Z^N}{1-Z}\right)_{m,1} = \left(\frac{1}{1-Z}\right)_{m,1}, \quad (17)$$

which implies that

$$\lim_{N\to\infty}\frac{E_N-E_0}{N} = k^2R^2 - \frac{k^2R\sin(4R\pi)}{4\pi}. \quad (18)$$

The right side of Eq. (18) is a monotonically increasing function of R. So one can always enhance the asymptotical diffusion rate by increasing the ratio of the well width and the kicking field wavelength in this case. In fact, the procedure derived above can also be applied to Eq. (6), and the asymptotical diffusion rate $\lim_{N\to\infty}[(E_N-E_0)/N]$ for any kicking potential $V(x)$ is given by $(1/2\pi)\int_0^{\pi}(V')^2dx$ [note that the potential $V(x)$ is assumed to be differentiable]. For clarifying it, we substitute $P_m(N-1)=(Z^{N-1})_{m,1}$ into Eq. (6), where the particle is assumed to be in the ground state. Similarly, after iterating it, we have

$$E_N = E_0 + \frac{N}{2\pi}\int_0^{\pi}(V')^2 dx - \frac{1}{2\pi}\sum_{m=1}^{\infty}\int_0^{\pi}(V')^2\cos(2mx)dx\left(\frac{1-Z^N}{1-Z}\right)_{m,1}. \quad (19)$$

Since the third term on the right side of Eq. (19) converges to a finite value as $N\to\infty$ [here, without loss of generality, it is assumed that $1/(1-Z)$ is well defined], the asymptotical diffusion rate is given by $(1/2\pi)\int_0^{\pi}(V')^2dx$, which is very similar to the result obtained in Ref. [8] for the measurement-induced diffusion of the kicked rotor. This similarity reveals the asymptotical equivalence caused by frequent measurements of the kicked rotor and the kicked particle confined in an infinite well, although the former is a KAM system and the latter is a non-KAM system.

III. ENTANGLEMENT BETWEEN THE PARTICLE AND THE MEASURING APPARATUS

As mentioned in Ref. [19], the experimental realization of system (1) can be achieved by putting cold atoms in a quasi-one-dimensional quantum dot. The atoms are then driven by a periodically pulsed standing wave of light. Then, similar to the procedure discussed in Ref. [8], the projective measurement of the energy at the time t' after every kick can be schematized by associating an additional degree of freedom, such as a spin, with every energy eigenstate. This is easily achieved by adding the following Hamiltonian [34] to system (1):

$$H_{mea} = \frac{\pi\hbar}{2}\sum_{n,N}|n\rangle\langle n| \otimes \sigma_1^{(n,N)}\delta(t-NT-t'), \quad (20)$$

where $\sigma_1^{(n,N)}=|+\rangle_{(n,N)}\langle -|+|-\rangle_{(n,N)}\langle +|$ is the first Pauli matrix of the spin recording the energy information in channel (n,N), where $|+\rangle_{(n,N)}$, $|-\rangle_{(n,N)}$ denote the spin-up and spin-down states of the spin $\sigma^{(n,N)}$, respectively. In fact, the evolution operator $U(NT+t'^{+},NT+t'^{-})$ caused by H_{mea} can be simply expressed as

$$U(NT+t'^{+},NT+t'^{-}) = \exp\left(-i\frac{\pi}{2}\sum_n |n\rangle\langle n| \otimes \sigma_1^{(n,N)}\right) = -i\sum_n |n\rangle\langle n| \otimes \sigma_1^{(n,N)}, \quad (21)$$

which is actually a generalized d-dimensional controlled-NOT gate operation. The controlled-NOT gate operation can be regarded as an entangler. We assume that all of the spins are initially in the spin-down states. It is not difficult to demonstrate that the total Hamiltonian of (1) plus H_{mea} can result in the evolution governed by Eq. (2) of the reduced density matrix of the measured system. In order to verify it, we need only insert $U[(N-1)T+t'^{+},(N-1)T+t'^{-}]$ into the time evolution operator generated by the Hamiltonian (1)

$$\rho_{NT+t'^{-}} = \mathrm{Tr}_{\{\sigma^{(N-1)}\}}\Big(\sum_{n,m} U_1|n\rangle\langle n|\rho_{(N-1)T+t'^{-}}|m\rangle\langle m|U_1^{\dagger} \otimes \sigma_1^{(n,N-1)}\bigotimes_j|-\rangle_{(j,N-1)}\langle -|\sigma_1^{(m,N-1)}\Big) = \sum_n U_1|n\rangle\langle n|\rho_{(N-1)T+t'^{-}}|n\rangle\langle n|U_1^{\dagger} \quad (22)$$

where $U_1=U_{free}(t')U_{kick}U_{free}(T-t')$, and $\rho_{NT+t'^{-}}$ is the reduced density matrix of the particle at the time $NT+t'^{-}$. In Eq. (22), $\mathrm{Tr}_{\{\sigma^{(N-1)}\}}$ denotes the tracing over all of the spins labeled by $\sigma^{(i,N-1)}$ $(i=1,2,\ldots)$. Equation (22) implies that the coupling between the system of interest and the apparatus causes the total system to become an entangled state and the reduced density matrix of the particle is completely projected on the energy eigenstates. Iterating Eq. (22), we obtain

$$\rho_{NT+t'^{-}} = \tilde{\mathcal{L}}^N\rho_{t'^{-}},$$

$$\tilde{\mathcal{L}}\rho = \sum_n U_1|n\rangle\langle n|\rho|n\rangle\langle n|U_1^{\dagger}, \quad (23)$$

which is exactly the expression of Eq. (2). As mentioned above, the system of interest and the measuring apparatus become an entangled state in this situation. It would be very interesting to investigate how the inherent quantum chaos affects the entanglement between the particle and measuring apparatus. In the following, we confine our discussions in the bipartite entanglement between the particle and all of the spins, and the partial entanglement between the particle and the partial spins $\sigma^{(N)}$. It is found that the bipartite entanglement and the partial entanglement between pairs exhibit two distinct dynamical behaviors. For characterizing the entanglement, we adopt the relative entropy of entanglement defined by $E_r(\rho)=\min_{\varrho\in\Omega}S(\rho\|\varrho)$ [35], where Ω is the set of all disentangled states, and $S(\rho\|\varrho)\equiv\mathrm{Tr}[\rho(\log_2\rho-\log_2\varrho)]$ is the quantum relative entropy. The relative entropy of entanglement is a good measure of quantum entanglement, and it reduces to the von Neumann entropy of the reduced density matrix of either subsystem for pure states. So the bipartite entanglement between the particle and all of the spins can be characterized by the von Neumann entropy of the reduced density matrix of the particle if both the particle and the measuring apparatus are initially in a pure state. At the time $t=NT+t'^{+}$, the bipartite entanglement can be expressed as

$$S_V(N) = -\mathrm{Tr}(\rho_{NT+t'^{+}}\log_2\rho_{NT+t'^{+}}) = -\sum_{i=1}^{\infty}P_i(N)\log_2 P_i(N). \quad (24)$$

In Fig. 4, the von Neumann entropy $S_V(N)$ in the case with $V(x)=k\cos(2Rx)$ is plotted for three different values of R. It is shown that the entanglement grows with the kicking steps. The growth of entanglement exhibits considerable R dependence, and increasing R to enhance classical diffusion can lead to significantly larger entanglement for the same field strengths in this case. By tracing the degrees of freedom of the additional spins, the particle and the spins $\sigma^{(N)}$ at time

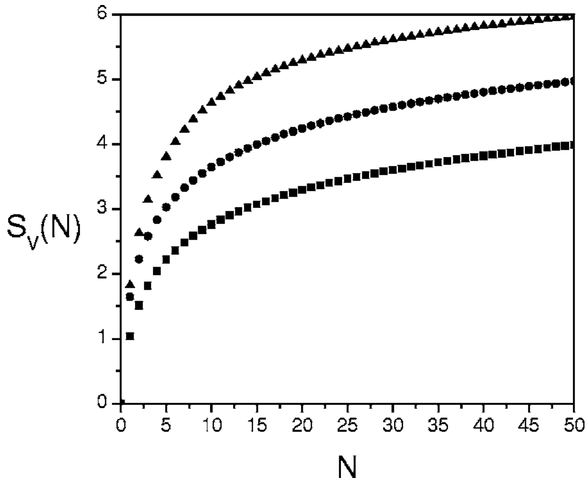

FIG. 4. The von Neumann entropy $S_V(N)$ of the reduced density matrix of the particle at the time $NT+t'^{+}$ is plotted as a function of N for different values of R. The particle is initially in the ground state and the kicking potential $V(x)=k\cos(2Rx)$ is chosen as an illustration; $k=\hbar$. Solid square, $R=\pi/4$; solid circle, $R=\pi/2$; solid triangle, $R=\pi$.

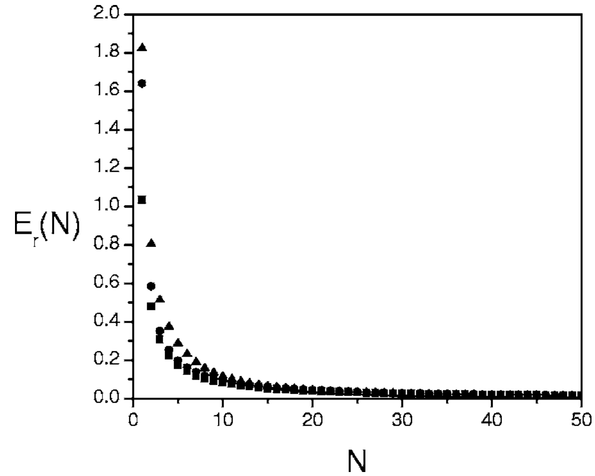

FIG. 5. The pairwise entanglement characterized by the relative entropy of entanglement $E_r(N)$ of the subsystem containing the particle and the partial spins $\sigma^{(N)}$ at the time $NT+t'^{+}$ is plotted as a function of N for different values of R. The particle is initially in the ground state, and the spins are initially in the spin-down states. $V(x)=\hbar\cos(2Rx)$; solid square, $R=\pi/4$; solid circle, $R=\pi/2$; solid triangle, $R=\pi$.

$t=NT+t'^{+}$ are in a maximally correlated state [36], which is given by

$$\rho_{sm}=\sum_{n,m}|n\rangle\langle n|U_1\rho_{(N-1)T+t'^{+}}U_1^{\dagger}|m\rangle\langle m| \otimes \sigma_1^{(n,N)}(\bigotimes_j|-\rangle_{(j,N)}\langle-|)\sigma_1^{(m,N)}. \quad (25)$$

The partial entanglement quantified by the relative entropy of entanglement between the particle and the partial spins $\sigma^{(N)}$ at the time $t=NT+t'^{+}$ can be calculated as

$$E_r(N)=-\sum_{n=1}^{\infty}P_n(N)\log_2 P_n(N)+\mathrm{Tr}(\rho_{sm}\log_2\rho_{sm}) = -\sum_{n=1}^{\infty}P_n(N)\log_2 P_n(N)+\sum_{n=1}^{\infty}P_n(N-1)\log_2 P_n(N-1). \quad (26)$$

Equation (26) implies that the partial entanglement between pairs $E_r(N)$ exactly equals $S_V(N)-S_V(N-1)$, i.e., the increment of the bipartite entanglement. So the bipartite entanglement can also be rewritten as $S_V(N)=\Sigma_{n=1}^{N}E_r(n)$, which means that the bipartite entanglement $S_V(N)$ equals the area of the zone below the discrete curve of $E_r(n)$ $(n=1,\ldots,N)$. In Fig. 5, $E_r(N)$ is displayed for three different values of R. Contrary to the increase of the bipartite entanglement, we can see that E_r first achieves its maximal value at $t=T+t'^{+}$, then rapidly decreases with N. There exist two main factors that could affect the partial entanglement between pairs. One is the mixedness (ordinarily characterized by the von Neumann entropy or the linear entropy), and the other is the coherence of the particle (characterized by the nondiagonal elements of density matrix). The increase of coherence can enhance the entanglement, while the increase of mixedness usually causes a decrease of entanglement. So, in this case, diminution of partial entanglement between the particle and the partial spins $\sigma^{(N)}$ may be owing to the fact that the measurement-assisted quantum diffusion step by step increases the mixedness of the system of interest, which plays a more prominent role in the partial entanglement between pairs than the other factor.

At the end of this section, some natural questions arise: How does the bipartite entanglement behave asymptotically? What is the relation between the asymptotical bipartite entanglement and the asymptotical diffusion rate? In the present study, these are still the open questions and need to be investigated in future work.

IV. CONCLUSIONS

In this paper, we study the diffusion and entanglement in the system of a kicked particle in an infinite square well under frequent measurements. It is shown that the measurement-assisted diffusion rate can be constant for a large class of kicking fields. The asymptotical diffusion rate is also given as $(1/2\pi)\int_0^{\pi}(V')^2dx$. Then we investigate the bipartite entanglement between the particle and the whole measuring apparatus and partial entanglement between the particle and the partial spins. It is found that there exist two distinct dynamical behaviors of entanglement. The bipartite entanglement grows with the kicking steps and it gains a larger value for a more chaotic system. However, the partial entanglement between the system of interest and the partial spins of the measuring apparatus decreases with increasing number of kicking steps. For a more chaotic system, the bipartite entanglement grows at a higher rate at short times. The increment of the bipartite entanglement between two coterminous measurements equals the partial entanglement between pairs, which asymptotically tends to zero. However, this does not imply that the bipartite entanglement asymptotically reaches saturation. Whether or not there exists a saturation of bipartite entanglement in such a joint system (in which the dimension of the Hilbert space is infinite) is still an open question in this paper. Nevertheless, we can conclude that the diffusion of the system and the bipartite entanglement of the system and the whole measuring apparatus exhibit cooperation in the case with a small value of intensity of the kicking field, namely, the diffusion enlarges the effective Hilbert space to be entangled with the measuring apparatus and hence enlarges the bipartite entanglement, and conversely the bipartite entanglement destroys the local coherence of the system of interest and enhances the diffusion.

It has been mentioned that the quantum dot might be a suitable candidate for experimental realization of the system (1). One of the key steps is the implementation of repeat measurements of the energy. It is unambiguous that direct frequent measurement of the energy of the atom confined in the quantum dot is not a trivial task. In the past few years, much attention has been paid to quantum measurement schemes in quantum dots such as the quantum point contact and so on. In some situations, the quantum point contact can cause dephasing of the quantum state. We briefly discuss the system in the presence of the particular dephasing that is described by the following master equation:

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H(t),\rho] - \frac{\gamma(t)}{2}[p^2,[p^2,\rho]], \qquad (27)$$

where $H(t)$ is given by Eq. (1) and $\gamma(t)$ is a constant γ_0 or $\sum_{l=-\infty}^{\infty}\varepsilon_0\delta(t-lT-t')$. It is not difficult to verify that two cases are equivalent if $\gamma_0 T=\varepsilon_0$. The evolving density operator in the latter case is completely reduced to the one described by Eq. (2) when $\varepsilon_0\to\infty$. In the case with $\gamma_0 T\gg 1$, the evolution of the density operator described by Eq. (27) closely approximates to Eq. (2). Therefore, to some extent, one can also regard the repetitive measurement of energy as a "noncoherent energy-preserving kick." Though the influence of the relaxation process on the dynamics needs to be taken into account when the numerical simulation of the realistic situation is executed. Usually, the dephasing time is much shorter than the relaxation time in quantum dots. This fact provides us with the possibility of experimentally studying the measurement-induced dynamical behavior of system (1) via the dephasing process. Recently, some authors have studied the effects of measurements on dynamical localization in the kicked rotor model simulated on a quantum computer [37]. It was shown that localization can be preserved for repeated single-qubit measurements. A transition from a localized to a delocalized phase was revealed, which depends on the system parameters and the choice of the measured qubit. To a certain extent, the local measurement on one qubit can be regarded as a kind of partial dephasing in the kicked rotor according to the encoding scheme. It could be conjectured that the different choices of the measured qubit in the quantum computer scheme have similar effects to those of different dephasing coefficients on the dynamical behavior of simulated quantum chaotic systems. Therefore, it is also very interesting to investigate whether certain kinds of transition can occur in a system of a kicked particle inside an infinite square well when the dephasing coefficient varies.

Recently, the kicked Bose-Einstein condensate (BEC) has attracted much attention. Some authors have investigated the quantum resonance and antiresonance of a kicked BEC confined in a one-dimensional box [38]. The results presented in this paper can be generalized to those systems. It is expected that the frequent measurement of the energy or dephasing mechanism may significantly change their dynamical behaviors. If the nonlinear interatomic interaction is ignored, they are reduced to the present case. For the situation with a very small nonlinear interatomic interaction, we can adopt a similar approximation procedure to that in Ref. [38] to investigate the effect of the nonlinear interatomic interaction on measurement-induced diffusion.

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